with τ in equation (4). The propagation speed of the front as described by the theory is in fair agreement with the numerical results. It is demonstrated that, within the range of its basic assumptions, the model captures the qualitative essentials of the propagating temperature front.

Acknowledgement—This work was conducted while the author was on the faculty of Clarkson University, Potsdam, New York.

REFERENCES

1. J. Patterson and J. Imberger, Unsteady natural convection in a rectangular cavity, *J. Fluid Mech.* **100**, 65–86 (198).
2. R. Yewell, D. Poulikakos and A. Bejan, Transient natural convection experiments in cavities, *Trans. Am. Soc. mech. Engrs*, Series C, *J. Heat Transfer* **104**, 533–538 (1982).
3. J. Patterson, On the existence of an oscillatory approach to steady natural convection in cavities, *Trans. Am. Soc. mech. Engrs*, Series C, *J. Heat Transfer* **106**, 104–108 (1984).
4. J. M. Hyun, Transient process of thermally stratifying an initially homogeneous fluid in an enclosure, *Int. J. Heat Mass Transfer* **2**, 1936–1938 (1984).
5. J. M. Hyun, Thermally-forced stratification build-up in an initially isothermal contained fluid, *J. phys. Soc. Japan* **54**, 942–949 (1985).
6. . Rahm and G. Walin, On thermal convection in stratified fluids, *Geophys. Astrophys. Fluid Dynam.* **13**, 51–65 (1979).
7. L. Rahm, A note on the heat-up of an initially isothermal fluid, *Mathl Modell.* **6**, 19–30 (1985).
8. G. Walin, Contained non-homogeneous flow under gravity or how to stratify a fluid in the laboratory, *J. Fluid Mech.* **48**, 647–672 (1971).
9. E. M. Sparrow and J. L. Gregg, Laminar free convection from a vertical plate with uniform surface heat flux, *Trans. Am. Soc. mech. Engrs* **78**, 435–440 (1956).
10. A. Warn-Varnas, W. W. Fowlis, S. Piacsek and S. M. Lee, Numerical solutions and laser-Doppler measurements of spin-up, *J. Fluid Mech.* **85**, 609–639 (1978).

Int. J. Heat Mass Transfer. Vol. 29, No. 3, pp. 501–503, 1986
Printed in Great Britain
0017–9310/86 $3.00 + 0.00

Radiant-interchange configuration factors between a disk and a segment of a parallel concentric disk

SUI LIN,* PAI-MOW LEE,† JOSEPH C. Y. WANG,‡ WEI-LIANG DAI* and YOU-SHI LOU§

* Department of Mechanical Engineering, ‡ Centre for Building Studies and § Department of Mathematics, Concordia University, Montreal, Canada H3G 1M8

† School of Engineering, Lakehead University, Thunderbay, Canada P7B 5E1

(*Received* 18 *June* 1985 *and in final form* 30 *September* 1985)

1. INTRODUCTION

THE SOLUTION of practical thermal radiation problems depends frequently on the availability of interchange configuration factors. The interchange configuration factors for many practical geometries have been presented [1–6]. The important group of geometries which were not presented is the case of a disk radiating to a segment of a parallel concentric disk. The purpose of this paper is to present the results of the configuration factors of this group of geometries.

2. DETERMINATION OF THE CONFIGURATION FACTORS

For the determination of the configuration factors for radiant interchange between a disk and a segment of a parallel concentric disk, a schematic diagram, Fig. 1, shows the coordinate system for the relative position of the disk and the segment.

It is well known that the configuration factors, $F_{A_1-A_2}$, under the assumption that the magnitude and surface distribution of the radiosity is uniform over A_1, can be expressed by

$$F_{A_1-A_2} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi r^2}\, dA_1\, dA_2 \qquad (1)$$

where β_1 and β_2 are the angles formed by the normals of the elements dA_1 and dA_2 and the connecting line between the elements dA_1 and dA_2, as shown in Fig. 1. r represents the length of the connecting line. The contour of the segment can be expressed by

$$y = \pm\sqrt{a^2 - x^2}. \qquad (2)$$

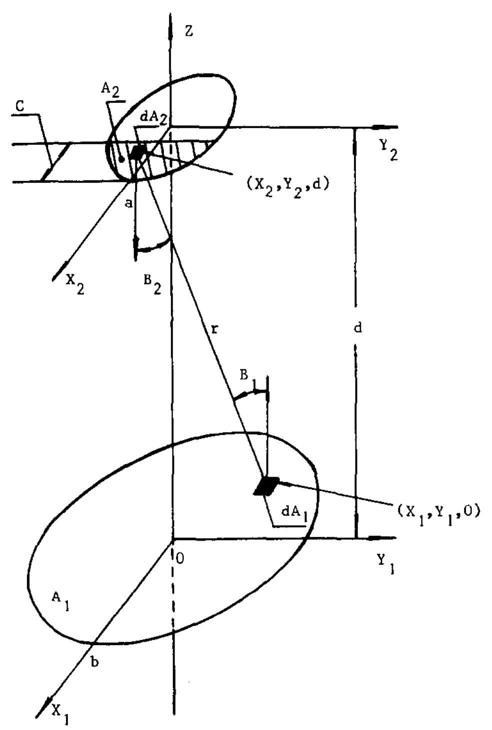

FIG. 1. Geometric configuration for radiant interchange between a disk and a segment of a parallel concentric disk.

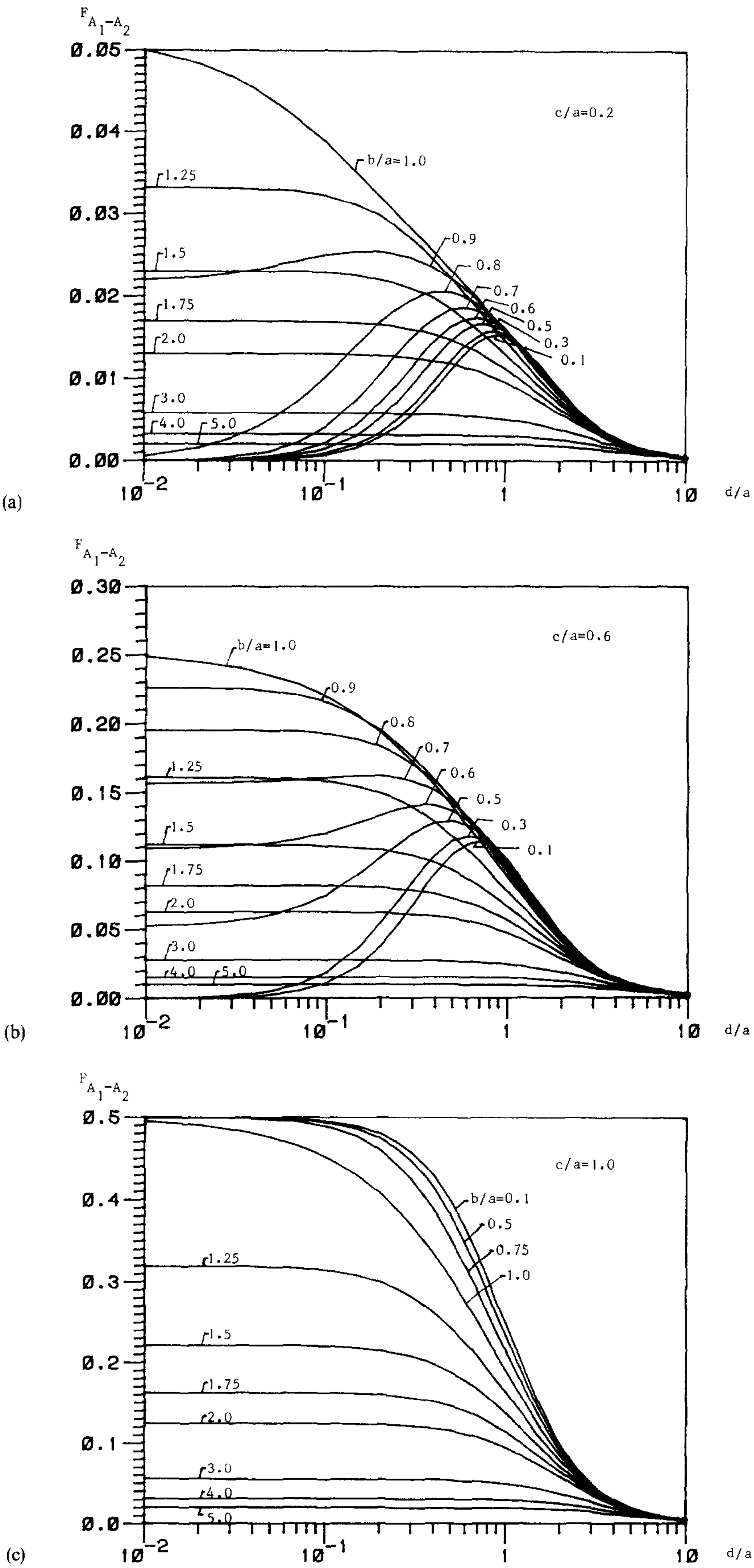

FIG. 2. Configuration factors, $F_{A_1-A_2}$, for radiant interchange between a disk and a segment of a parallel concentric disk, as a function of d/a with b/a as a parameter. (a) For $c/a = 0.2$; (b) for $c/a = 0.6$; (c) for $c/a = 1.0$.

The angles β_1 and β_2 can be obtained as

$$\cos\beta_1 = \cos\beta_2 = \frac{d}{r} \tag{3}$$

where

$$r = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+d^2}. \tag{4}$$

The areas of the elements dA_1 and dA_2 can be expressed as

$$dA_1 = dx_1\,dy_1 \tag{5}$$

$$dA_2 = dx_2\,dy_2 \tag{6}$$

and the total surface area of the disk is

$$A_1 = \pi b^2. \tag{7}$$

Substituting equations (2)–(7) into (1) gives

$$F_{A_1-A_2} = \frac{d^2}{\pi A_1}\int_{-b}^{b} dx_1 \int_{a-c}^{a} dx_2 \int_{-\sqrt{b^2-x_1^2}}^{\sqrt{b^2-x_1^2}} dy_1 \times \int_{-\sqrt{a^2-x_2^2}}^{\sqrt{a^2-x_2^2}} \frac{dy_2}{[(x_1-x_2)^2+(y_1-y_2)^2+d^2]^2}. \tag{8}$$

Equation (8) can be reduced to a double definite integration with constant lower and upper limits as

$$F_{A_1-A_2} = \frac{d^2}{\pi A_1}\int_{-b}^{b} dx_1 \times \int_{a-c}^{a} \frac{1}{X^3}\left[Y_1 \mathrm{tg}^{-1}\left(\frac{Y_1}{X}\right) - Y_2 \mathrm{tg}^{-1}\left(\frac{Y_2}{X}\right)\right] dx_2 \tag{9}$$

where

$$X = \sqrt{d^2+(x_1-x_2)^2}$$

$$Y_1 = \sqrt{a^2-x_2^2} + \sqrt{b^2-x_1^2}$$

$$Y_2 = \sqrt{a^2-x_2^2} - \sqrt{b^2-x_1^2}.$$

Equation (9) was numerically integrated; Figs. 2(a)–(c) show the configuration factors, $F_{A_1-A_2}$, as a function of d/a with b/a as a parameter for $c/a = 0.2$, 0.6 and 1.0, respectively. It is of interest to note that, except for the case of $c/a = 1.0$, maximum values of the radiant interchange configuration factors could be obtained by adjusting the distance between the disk and the segment, for the cases of $b/a < 1.0$. For $b/a < 1.0$, it indicates that the diameter of the disk is smaller than the diameter of the segment disk. We consider the case of $b/a = 0.5$ (the diameter of the disk is only one half of the diameter of the segment disk) and $c/a = 0.6$. When the disk is far away from the segment, say $d/a = 10$, the configuration factor $F_{A_1-A_2}$ is about 0.003. However, when the disk is very close to the segment, because they can hardly see each other, the configuration factor $F_{A_1-A_2}$ is close to zero. Especially for $d/a = 0$, $F_{A_1-A_2}$ is equal to zero, because they can no longer see each other. Therefore the configuration factor, $F_{A_1-A_2}$, has maximum values as shown in Figs. 2(a)–(c).

3. ERROR ESTIMATION

In order to evaluate the accuracy of the numerical integration, $F_{A_1-A_2}$ calculated for the case of $c/a = 1.0$ and $b/a = 1.0$ (the segment under this consideration becomes a half disk having a diameter as the same as that of the disk) was compared with that obtained from the solution of the radiant-interchange between two parallel disks [1–6]. It has been tested for the case of $0.01 \leqslant d/a \leqslant 10.0$ that both results agree to six places of decimals for the case of $d/a < 5.0$, and to five places of decimals for $d/a \geqslant 5.0$.

Acknowledgements—The present work was supported by the Natural Science and Engineering Research Council of Canada, under Grant No. A7929.

REFERENCES

1. R. Siegel and J. R. Howell, *Thermal Radiation Heat Transfer*, 2nd edn, pp. 782–831. McGraw-Hill, New York (1981).
2. E. M. Sparrow and R. D. Cess, *Radiation Heat Transfer*, pp. 339–349. McGraw-Hill, New York (1978).
3. H. C. Hottel and A. F. Sarofim, *Radiative Transfer*, pp. 25–71. McGraw-Hill, New York (1967).
4. H. Y. Wang, *Handbook of Essential Formula and Data on Heat Transfer for Engineers*, pp. 100–120. Longman, New York (1977).
5. J. A. Stevenson and J. C. Grafton, Radiation heat transfer analysis for space vehicles, SID 62–91, North American Aviation, ASD TR. 61–119, pt. 1 (December 1961).
6. J. R. Howell, *A Catalogue of Radiation Configuration Factors*, pp. 89–242. McGraw-Hill, New York (1982).